Question 1 **[Marks 20]**

(i) The center portion of the rubber balloon has a diameter of d_1 = 100 mm. If the air pressure causes the balloon's diameter to become d_2 = 125mm, what is the average normal strain in the rubber? **[Marks 4]**

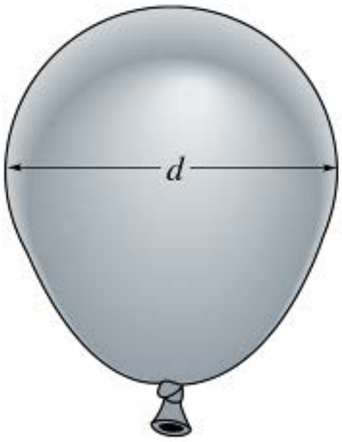

Figure 1(i)

(ii) A rectangular plate is deformed due to applies shear loading into the dashed shape shown in figure 1(ii). Determine (a) the average normal strain along the side *AB*, and (b) the average shear strain in the plate. **[Marks 6]**

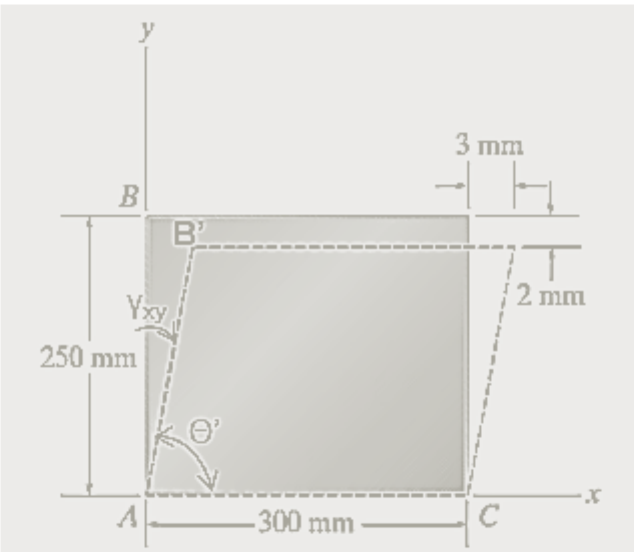

Figure 1(ii)

(iii) The bolt shown in figure 1(iii) is made of an aluminum alloy and is tightened so it compresses a cylindrical tube made of a magnesium alloy. The tube has an outer radius of 10 mm, and both the inner radius of the tube and the radius of the bolt are 5 mm. The washers at the top and bottom of the tube are considered to be rigid and have a negligible thickness. Initially the nut is hand-tightened slightly; then, using a wrench, the nut is further tightened one-half turn. If the bolt has 20 threads per 20mm (one turn, the nut advances 20/20=1mm along the bolt), determine the stress in the bolt. Take Young's modulus E_{al}=75GPa, E_{mag} = 45GPa **[Marks 10]**

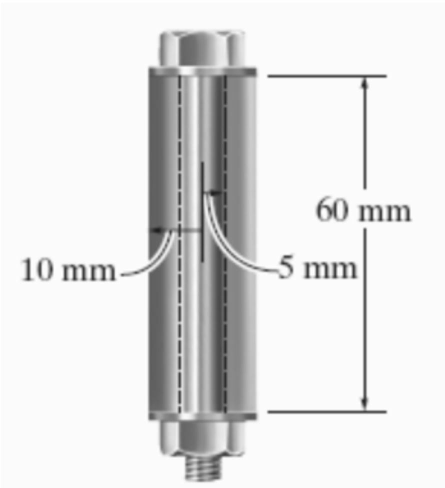

Figure 1 (iii)

Question 2 **[Marks 20]**

(i) A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends as shown in figure 2(i). Portion *AB* is made of steel $(E_s = 200\text{ GPa}, \alpha_s = 11.7 \times 10^{-6}/°\text{C})$ and portion *BC* is made of brass $(E_b = 105\text{ GPa}, \ \alpha_b = 20.9 \times 10^{-6}/°\text{C})$. Knowing that the rod is initially unstressed, determine the compressive force induced in *ABC* when there is a temperature rise of $50\,°\text{C}$. **[Marls 10]**

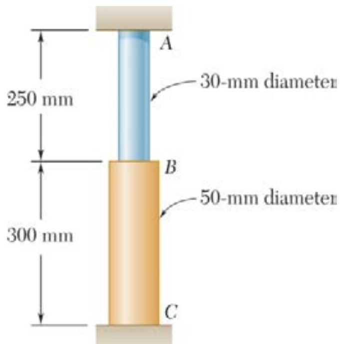

Figure 2(i)

(ii) Determine minimum diameter, d, of the bolts for the connections shown in figure(ii) A and figure(ii) B. Given the shear yield stress, $\tau_Y = 250$MPa, and factor of safety, S_f=2.5. **[Marks 10]**

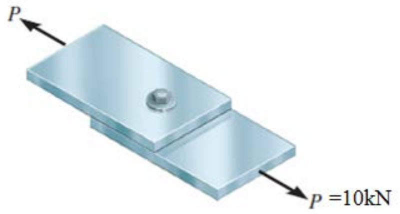

Figure 2(ii) A

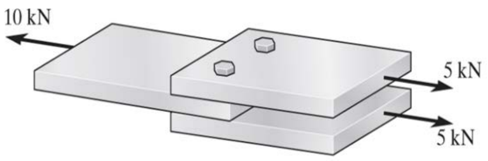

Figure 2(ii) B

Question 3 **[Marks 20]**

(i) The solid shaft shown in figure 3(i) is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

[Marks 8]

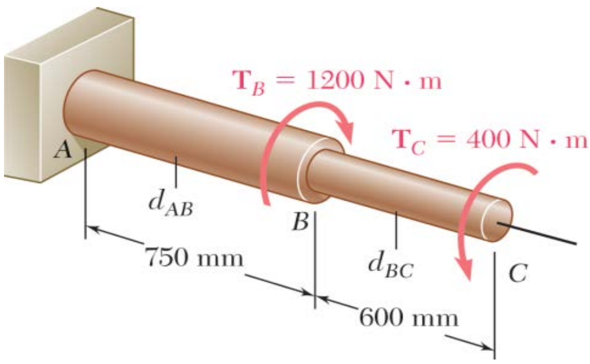

Figure 3(i)

(ii) Two vertical forces are applied to a beam of the cross section shown in figure 3(ii). Determine the maximum tensile and compressive stresses in portion *BC* of the beam. **[Marks 12]**

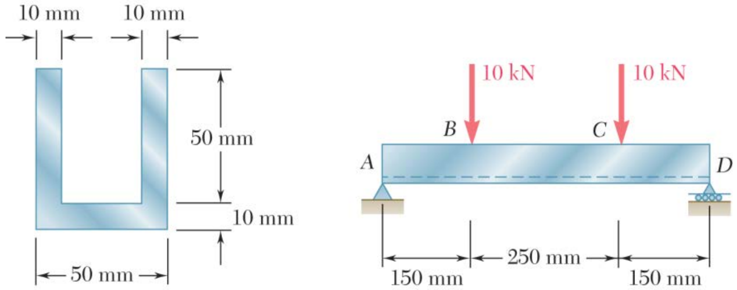

Figure 3(ii)

Question 4 **[Marks 20]**

(i) Draw the shear and moment diagrams for a beam with a uniform distributed load (UDL) of intensity w= 6kN/m; L=3m as shown in figure 4(i). What is magnitude and location of maximum bending moment? **[Marks 10]**

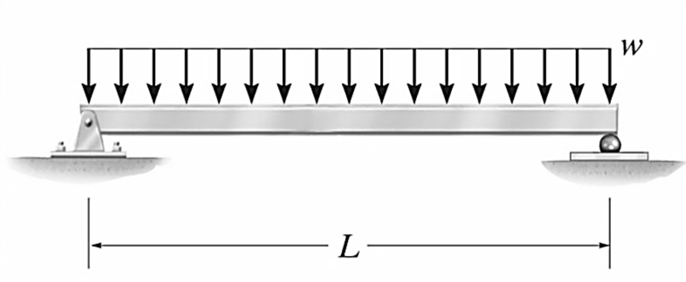

Figure 4(i)

(ii) For the state of plane stress shown in figure 4(ii), determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses. **[Marks 10]**

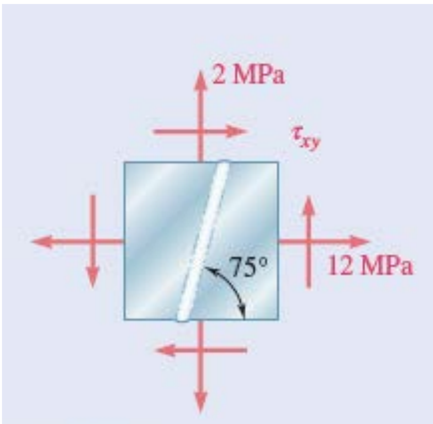

Figure 4(ii)

Question 5 **[Marks 20]**

(i) A steel plate under tension as shown in Figure 5(i) has two fixed supports. Your manager has asked you to model the plate with simply supported boundary conditions. What changes would you recommend in the boundary conditions to represent the true in-plane behaviour of simply supported plate under tension?

[Marks 5]

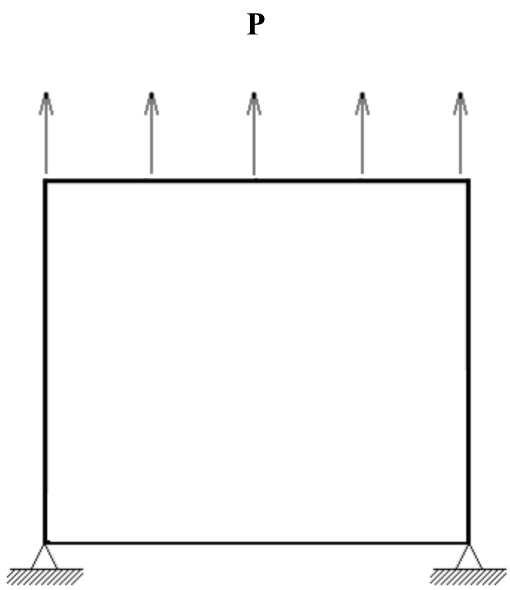

Figure 5(i)

(ii) Show the Poison's effect (in-plane deformation) of: (1) the plate shown in figure 5(i); (2) the plate after recommending changes in the boundary conditions in part (i) **[Marks 5]**

(iii) What is over-stiffening and under-stiffening of a structure. How can you avoid both while constructing FEA model? **[Marks 5]**

(iv) How can you use symmetry to obtain quick and accurate results? **[Marks 5]**

IMPORTANT FORMULAE

ALL SYMBOLS HAVE THEIR USUAL MEANING

Normal stress and strain

$$\sigma = \frac{F}{A};\ \varepsilon = \frac{\Delta L}{L};\ \sigma = E\varepsilon$$

$$\varepsilon_T = \alpha(T_f - T_o)$$

$$\varepsilon_{lat} = -\nu\varepsilon_{long}$$

Simple Bending

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$M_{\max} = \sigma_{\max} Z$$

$$I = \frac{bh^3}{12} \qquad I = \frac{\pi d^4}{64} \qquad I = \frac{bh^3}{36}$$

Torsion of Shafts

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

$$J = \frac{\pi d^4}{32},\quad J = \frac{\pi(D^4 - d^4)}{32}$$

$$P = \varpi T = \frac{2\pi NT}{60}$$

Stress-strain relationships

$$\sigma = \sigma_n(1 + \varepsilon_n)$$

$$\varepsilon = \ln(1 + \varepsilon_n)$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

Stress transformation

$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_s = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{\max,\min} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2]}$$

$$\tau_{\max} = \pm\frac{1}{2}\sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2]}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_{\tau_{\max}} = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Strain transformation

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_s}{2} = -\frac{1}{2}(\varepsilon_x - \varepsilon_y)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{\max,\min} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{[(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2]}$$

$$\gamma_{\max} = \sqrt{[(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2]}$$

$$\tan 2\varphi_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Thin cylinder

$$\sigma_L = \frac{Pr}{2t} = \frac{pD}{4t} \; ; \qquad \sigma_C = \frac{Pr}{t} = \frac{pD}{2t}$$

Thick cylinder

$$\sigma_r = A - \frac{B}{r^2} \; ; \qquad \sigma_C = A + \frac{B}{r^2}$$

$$A = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} \; ; \qquad B = \frac{(P_i - P_o) r_i^2 r_o^2}{r_o^2 - r_i^2}$$

Yield criteria

Shear strain energy theory: $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$

Total strain energy theory**:** $(\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2) - 2\nu(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3) = \sigma_y^2$

Maximum shear stress theory: $\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_y}{2}$ or $|\sigma_1 - \sigma_3| = \sigma_y \; (\sigma_1 > \sigma_2 > \sigma_3)$

Maximum principal stress theory: $|\sigma_1| = \sigma_y$

Beam Deflection

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{EI}$$

$$\delta_{\max} = \frac{WL^3}{kEI}$$

$k = 3, cantilever, point\ load$

$k = 8,\ cantilever, uniformly\ disributed\ load$

$k = 48, simple\ supports, centre\ point\ load$

$k = 384\ simple\ supports, distributed\ load$